TRINITY ⚜ COLLEGE

Mathematics Specialist Units 1,2
Test 2   2017

Section 1   Calculator Free
Vectors

STUDENT'S NAME   *SOLUTIONS*
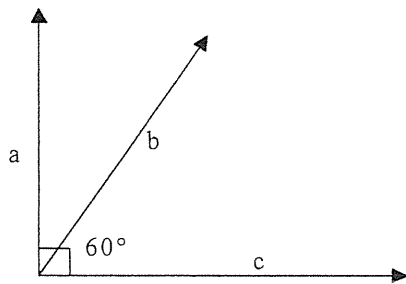
DATE: Friday 31 March   **TIME:** 28 minutes   **MARKS:** 28

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (4 marks)

Given the 3 vectors shown above and $|a| = 5$, $|b| = 6$ and $|c| = 4$, determine

(a) $a \bullet a$ [1]

$$5^2 = 25$$

(b) $a \bullet b$ [2]

$$5 \times 6 \times \cos 30°$$
$$= 15\sqrt{3}$$

(c) $a \bullet c$ [1]

$$0$$

2. (4 marks)

Determine all vectors of magnitude 5 that are perpendicular to $6i + 8j$.

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 8 \end{pmatrix} = 0$$

$$6a + 8b = 0$$

$$3a = -4b \qquad\qquad a = \frac{-4b}{3}$$

$$\sqrt{a^2 + b^2} = 5$$

$$\left(\frac{-4b}{3}\right)^2 + b^2 = 25$$

$$\frac{16b^2}{9} + b^2 = 25$$

$$16b^2 + 9b^2 = 225$$

$$25b^2 = 225$$

$$b^2 = 9$$

$$b = \pm 3$$

$$b = 3 \qquad a = -4$$
$$b = -3 \qquad a = 4$$

VECTORS $\begin{pmatrix} -4 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

3. (3 marks)

Determine the value/s of m if the vectors $a = \begin{pmatrix} m+1 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} m \\ m+1 \end{pmatrix}$ are perpendicular.

$$\begin{pmatrix} m+1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} m \\ m+1 \end{pmatrix} = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$m^2 + m - 2m - 2 = 0$$

$$m^2 - m - 2 = 0$$

$$(m-2)(m+1) = 0$$

$$m = 2, -1$$

4. (3 marks)

A and B have position vectors $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 8 \end{pmatrix}$ respectively. Determine the position vector of the point P that divides AB in the ration 3:2.

$$OP = OA + \frac{3}{5} AB$$

$$= \begin{pmatrix} 3 \\ -7 \end{pmatrix} + \frac{3}{5}\left(\begin{pmatrix} 8 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 3 \\ -7 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} 5 \\ 15 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

5. (10 marks)

Given the vectors $p = 16i - 2j$, $q = 15i + 8j$ and $r = 4i + bj$, determine the value of b in the following situations.

(a) p and r are parallel

$$\begin{pmatrix} 16 \\ -2 \end{pmatrix} = \lambda \begin{pmatrix} 4 \\ b \end{pmatrix}$$

[2]

$$16 = 4\lambda \qquad\qquad -2 = \lambda b$$
$$4 = \lambda \qquad\qquad -2 = 4b$$
$$\qquad\qquad\qquad -\frac{1}{2} = b$$

(b) p, q and r are collinear

[4]

$$\overrightarrow{pq} = \begin{pmatrix} 15 \\ 8 \end{pmatrix} - \begin{pmatrix} 16 \\ -2 \end{pmatrix} \qquad\qquad \overrightarrow{pr} = \begin{pmatrix} 4 \\ b \end{pmatrix} - \begin{pmatrix} 16 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 10 \end{pmatrix} \qquad\qquad\qquad = \begin{pmatrix} -12 \\ b+2 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 10 \end{pmatrix} = \lambda \begin{pmatrix} -12 \\ b+2 \end{pmatrix}$$

$$-1 = -12\lambda \qquad\qquad 10 = \lambda(b+2)$$
$$\frac{1}{12} = \lambda \qquad\qquad 10 = \frac{1}{12}(b+2)$$
$$\qquad\qquad\qquad 120 = b+2$$
$$\qquad\qquad\qquad 118 = b$$

(c) r is a unit vector

[1]

$$|r| > 1 \qquad\qquad NO\ SOLUTION$$

(d) $|r| = 7$

$$\sqrt{4^2 + b^2} = 7$$

[3]

$$16 + b^2 = 49$$
$$b^2 = 33$$
$$b = \pm\sqrt{33}$$

6. (4 marks)

The position vector of A is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. The vector of B relative to A and of B relative to C are $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ respectively. Determine the position vector of C.

$$_B r_A = r_B - r_A$$

$$\begin{pmatrix} -8 \\ 2 \end{pmatrix} = r_B - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} -8 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = r_B$$

$$\begin{pmatrix} -5 \\ 6 \end{pmatrix} = r_B$$

$$_B r_C = r_B - r_C$$

$$\begin{pmatrix} 5 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \end{pmatrix} - r_C$$

$$r_C = \begin{pmatrix} -5 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} -10 \\ -4 \end{pmatrix}$$

TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 2   2017

Section 2   Calculator Assumed
Vectors

STUDENT'S NAME _____ *SOLUTIONS* _____

DATE: Friday 31 March   TIME: 32 minutes   MARKS: 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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7.   (3 marks)

A coastguard boat is chasing a drug runner's boat and is directly behind it. The drug runner's boat is travelling with constant velocity of (16i − 12j) m/s. The coastguard is gaining on the drug runners at a constant 8 m/s.

Determine the velocity of the coastguard boat.

$$\left| \begin{array}{c} 16 \\ -12 \end{array} \right| = 20$$

$$\left| COASTGUARD \right| = 20 + 8$$
$$= 28 \ m/s$$

$$REQD \ VELOCITY = \frac{28(16i - 12j)}{20}$$

$$= \left(\begin{array}{c} 22.4 \\ -16.8 \end{array} \right)$$

8.   (5 marks)

To a motorcyclist travelling at 108 km/hr on a bearing of 137°, the wind appears to be coming from a bearing of 191° at 64 km/hr. Determine the true velocity of the wind.

$$_w\vec{U}_M = \vec{U}_w - \vec{U}_M$$

$$\begin{pmatrix} 12.21 \\ 62.82 \end{pmatrix} = \vec{U}_w - \begin{pmatrix} 73.66 \\ -78.99 \end{pmatrix}$$

$$\begin{pmatrix} 85.87 \\ -16.17 \end{pmatrix} = \vec{U}_w$$

$$\vec{U}_M = (r, \theta)$$
$$= (108, -47°)$$
$$= \begin{pmatrix} 73.66 \\ -78.99 \end{pmatrix}$$

$$_w\vec{U}_M = (r, \theta)$$
$$= (64, 79°)$$
$$= \begin{pmatrix} 12.21 \\ 62.82 \end{pmatrix}$$

9.   (4 marks)

An object moves with a constant velocity of (-2i - j) m/s. If the initial position of the object, with respect to the origin, is (18i - j) m, determine when the object is 28 m from the origin.

$$\underset{\sim}{r} = \begin{pmatrix} 18 \\ -1 \end{pmatrix} + t \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

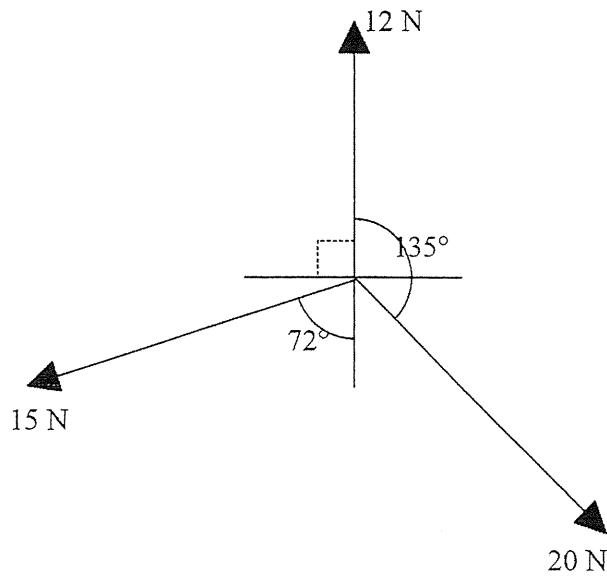$$\underset{\sim}{r} = \begin{pmatrix} 18 - 2t \\ -1 - t \end{pmatrix}$$

$$\left| \begin{matrix} 18 - 2t \\ -1 - t \end{matrix} \right| = 28$$

$$\sqrt{(18 - 2t)^2 + (-1 - t)^2} = 28$$

$$t = 18.87 \text{ sec}$$

10. (5 marks)

Determine the magnitude and the direction of the resultant of the three forces shown in the diagram below.

$$\left(12, 90°\right) + \left(20, -45°\right) + \left(15, -162°\right)$$

$$= \left(\begin{matrix} 0 \\ 12 \end{matrix}\right) + \left(\begin{matrix} 14.14 \\ -14.14 \end{matrix}\right) + \left(\begin{matrix} -14.26 \\ -4.64 \end{matrix}\right)$$

$$= \left(\begin{matrix} -0.12 \\ -6.78 \end{matrix}\right)$$
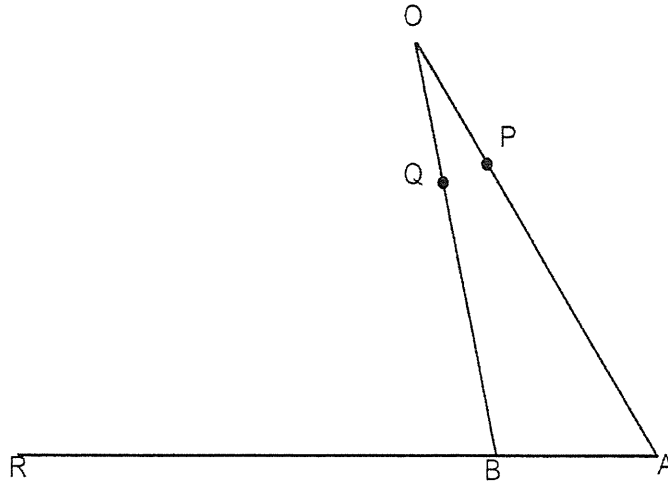
MAG   6.78 N

DIRECTION   -91.05°   (BEARING 181.05°)

11. (7 marks)

The origin O and the points P, Q, A, B and R are shown in the diagram below.
Also $\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OA}$, $\overrightarrow{AR} = 3\overrightarrow{AB}$, and $|\overrightarrow{OQ}| : |\overrightarrow{QB}| = 3:4$.

Let $\overrightarrow{OA} = \tilde{a}$ and $\overrightarrow{OB} = \tilde{b}$.

(a) Determine $\overrightarrow{PQ}$ in terms of $\tilde{a}$ and $\tilde{b}$.   [2]

$$OP = \frac{a}{3}$$

$$OQ = \frac{3b}{7}$$

$$PQ = \frac{3b}{7} - \frac{a}{3}$$

(b) Determine $\overrightarrow{PR}$ in terms of $\tilde{a}$ and $\tilde{b}$.   [2]

$$PA = \frac{2a}{3}$$

$$AB = b - a$$

$$AR = 3b - 3a$$

$$PR = PA - AR$$
$$= \frac{2a}{3} + 3b - 3a$$
$$= 3b - \frac{7a}{3}$$

(c) Show P, Q and R are collinear.   [3]

$$PQ = \frac{3b}{7} - \frac{a}{3}$$
$$= \frac{9b}{21} - \frac{7a}{21}$$
$$= \frac{1}{21}\left(9b - 7a\right)$$

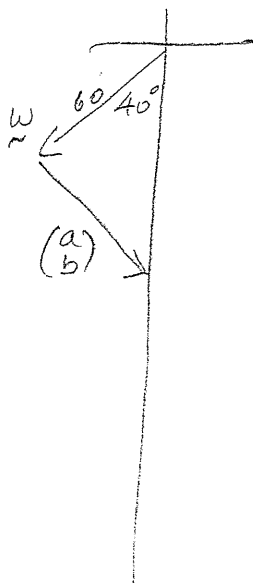$$PR = 3b - \frac{7a}{3}$$
$$= \frac{9b}{3} - \frac{7a}{3}$$
$$= \frac{1}{3}\left(9b - 7a\right)$$

∴ COLLINEAR

12.   (8 marks)

A pilot's destination is Perth from Bali (which is due South of Bali). The jet being flown can travel at 900 km/hr in still air. However, a 60 km/hr wind is blowing from a bearing of 40^0.

(a)   Determine the direction in which the pilot points the jet so that he can fly directly to Perth.   [4]

$$\left(60, -130^\circ\right) + \binom{a}{b} = \lambda \binom{0}{-3100}$$

$$\binom{-38\cdot57}{-45\cdot96} + \binom{a}{b} = \lambda \binom{0}{-3100}$$

$$a = 38\cdot57$$
$$b = 45\cdot96 - 3100\lambda$$

$$\sqrt{a^2 + b^2} = 900$$

$$\sqrt{38\cdot57^2 + (45\cdot96 - 3100\lambda)^2} = 900$$

$$\lambda = -0\cancel{28}\;,\; 0\cdot304$$

$$\binom{a}{b} = \binom{38\cdot57}{-896\cdot44}$$

(b)   How long will it take the pilot to fly to Perth given that it is 3100 km from Bali?   [2]

$$\frac{1}{0\cdot304} = 3\cdot29 \; HRS$$

(c)   At what actual speed does the jet fly?   [2]

$$\left| \binom{-38\cdot57}{-45\cdot96} + \binom{38\cdot57}{-896\cdot44} \right|$$

$$= \left| \binom{0}{942\cdot4} \right| \qquad 942\cdot4 \; km/hr$$